Vachhani sir's Technical Gems

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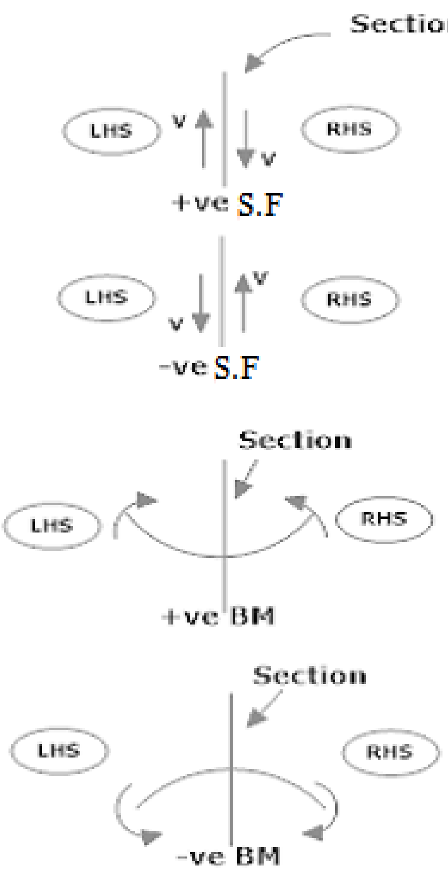
Chapter 10

Shearing force and Bending Moment Diagrams

A structural engineering framework consists of Beams and columns. Beams are subjected to shearing force and bending moment. It is therefore desirable to know the value of shearing force and Bending moment, all along the length of the beam.

What is Shearing force and Bending Moment and how to calculate and sketch the diagram for these.



Sign-Conventions

Shearing Force

It is defined to be the algebraic sum of all the vertical forces (including reactions), which are perpendicular to the horizontal axis of the beam on One Side of the section. Mathematical notation along with sign convention are given on the right hand side.

Bending Moment

It is defined to be the algebraic sum of moments of all the vertical forces (including reactions) on One Side of the Section.

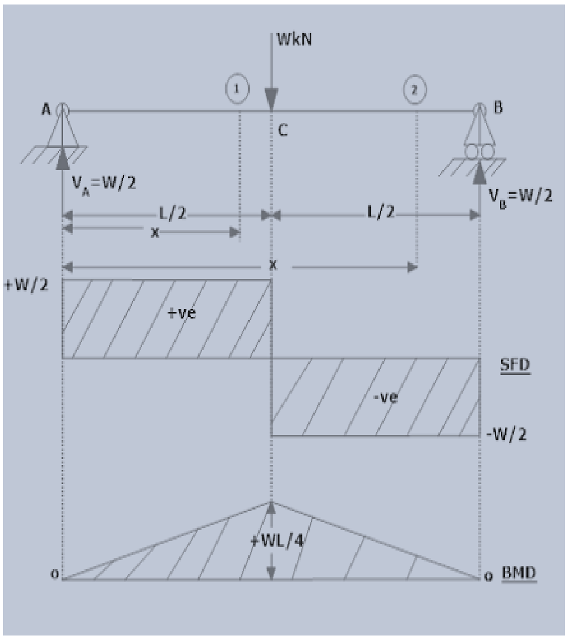
Standard Loading Problem

We will take first two problems of the simple beam and two problems of the cantilever beam. The results of these FOUR problems should be at the fingertips of all the students.

Standard loading Problem 1 (Proceed from Left-hand side)

1^{st} Step:
Calculate Reactions V_A & V_B. This has been already explained. Presently because of Symmetry

$$V_A = V_B = W/2 \uparrow$$



Shear force at any section x will be denoted by V_x and B.M will be denoted by M_x.

SFD (Shear Force Diagrams)
Equations & Calculations

AC, $V_x = + W/2$, $0 < x < L/2$ ---->(1)

CB, $V_x = + W/2 - W = - W/2$, $L/2 < x < L$ --->(2)

Eq.(1)

$x = 0$, $V_o = +W/2$, $x = L/2$, $V_L/2 = +W/2$,

Eq (2)

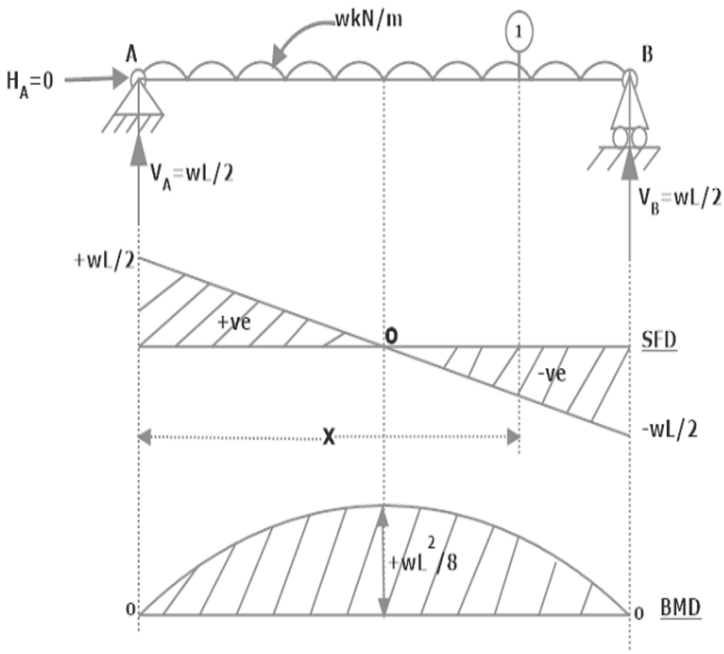
$x = L/2$, $V_{L/2} = -W/2$, $V_L = -W/2$

BMD (Bending Moment Diagram) Equations & Calculation

AC, $M_x = + W/2.x$, $0 < x < L/2$ ----(1) ,
$(M_A = M_o = 0,\ M_C = M_{L/2} = +WL/4)$

CB, $M_x = +W/2.x - W(x-L/2)$, $L/2 < x < L < x$,
$(M_c = M_{L/2} = +WL/4,\ M_B = M_L = 0)$

Standard loading problem 2



Because of Symmetry, Reactions

$V_A = V_B$ = Total Load/2 = $wL/2$

SFD Calculations

AB, $V_x = wL/2 - wx$ → (1)

L(Hence, there will be only one equation)

$V_o = wL/2$, $V_{L/2} = 0$, $V_L = -wL/2$

(1) is an equation to a straight line.

BMD Calculations

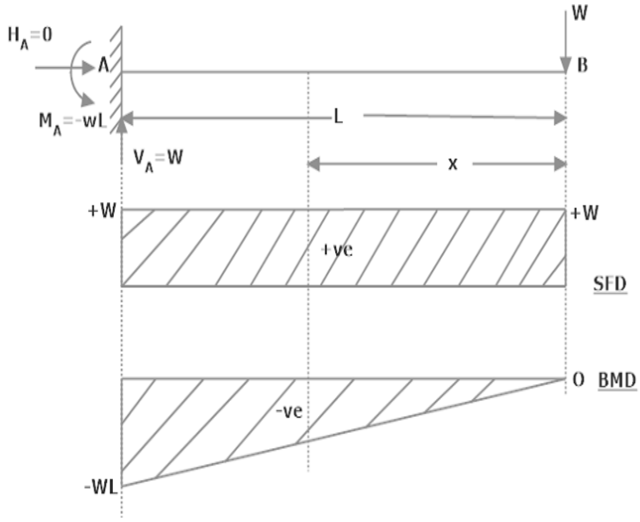
AB, $M_x = wL/2 .x - wx^2/2$ → (1)

Which is an equation to Parabola

$M_A = M_0 = 0$, $\underline{M_C = M_{L/2} = + wL^2/8}$, $M_B = M_L = 0$

It may be noted that Maximum BM occurs when SF=0

Standard Loading Problem3 Cantilever Beam



Reactions

ΣH=0, ----→+ve, Therefore +HA =0

ΣV=0,↑+ve, +V_A-W=0, Therefore,V_A =+W↑

Σ$M_{at\,A}$=0, ↷ +ve, +M_A + H_A x 0 + V_A x 0 + WL = 0
(Assume M_A +ve, i.e clockwise)

Therefore M_A= - WL

SFD, Proceeding from R.H.S

AB, V_x=+W -----→(1) (Equation to a straight line)

Please see RHS for Sign Conventions

Here x is measured from RHS

BA, M_x=-W.x ----→1, When x=0, M_o=M_B=0

When x=L, M_L=M_A=-WL

This equation is also an equation to a straight line.

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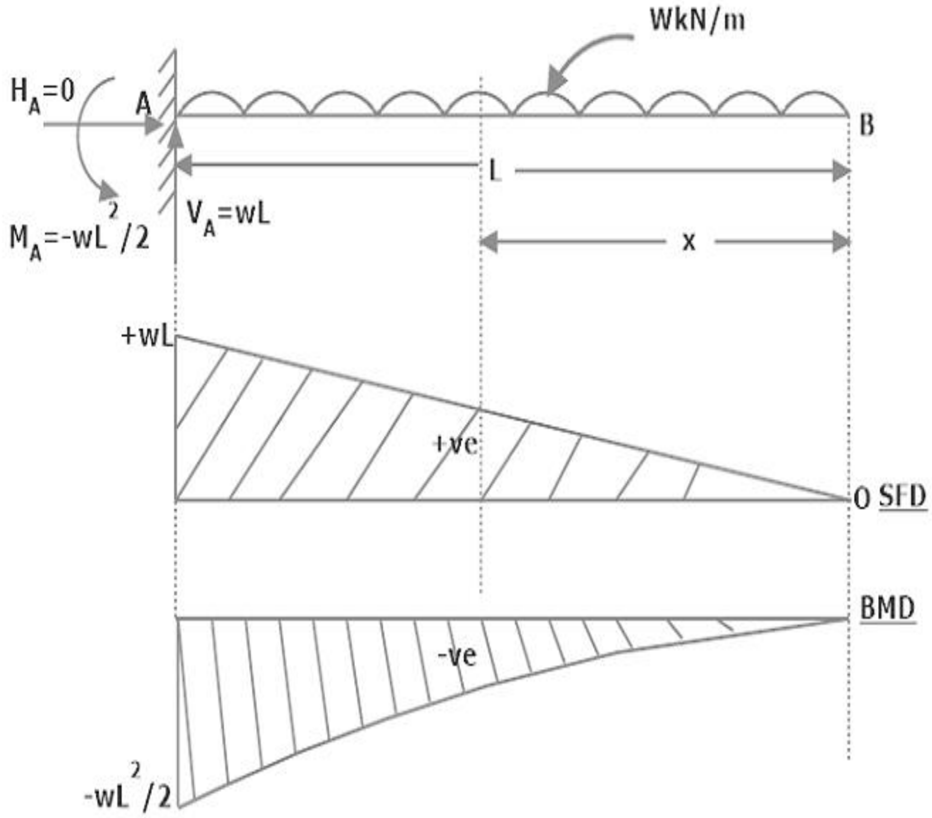
Standard Loading Problem-4



Reactions

$\Sigma H=0 \dashrightarrow +ve,$ $H_A=0$

$\Sigma V=0,$ $\uparrow +ve,$ $+V_A-wL=0,$ Therefore $V_A=wL$ $+\uparrow$

$\Sigma M_{atA}=0,$ $\curvearrowright$ $+ve,$

$+M_A$ (Assume Clockwise) $+H_A x0+V_A x0+wL^2/2=0$

Therefore $M_A= -wL^2/2$ kN.m

Proceeding from R.H.S.

Measure x from R.H.S.

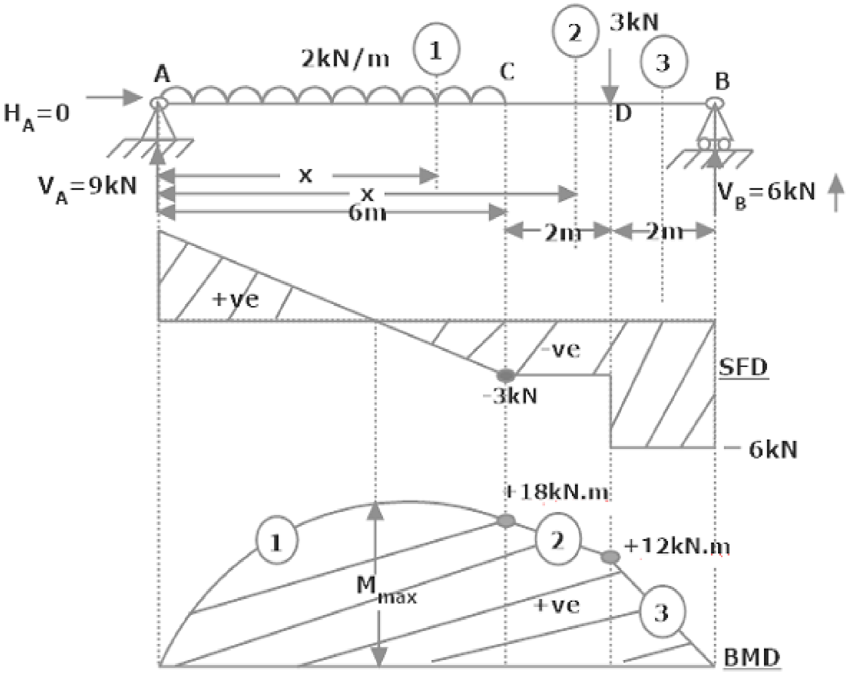
$V_x= +wx$ $\dashrightarrow$(1) For SF, $Vo=0, VL=+wL$

Equation to a straight line

$M_x= -wx.x/2=-wx2/2$ for BM, $M_0=0$, $M_L=-wL^2/2$ kN.m

Equation to a parabola. (Please see sign convention of R.H.S only).

Example 5 : Sketch SF and BM diagrams giving the values at principal locations at A,B,C and D.



<u>Solution:</u>

Reactions:

$\Sigma H=0$, ----→+ve $H_A=0$

$+V_A+V_B-2x6-3=0$, Therefore, $V_A+V_B=15$ -----→1

ΣMat A $=0$, ↻ +ve

$H_A \times 0+V_A \times 0+(2 \times 6)(6/2)+3 \times 8-V_B \times 10=0$

Therefore $V_B=+6kN$, From(1) we get, $V_A=+9kN$

<u>Shear Force Calculations and SFD</u>

AC, $V_x=+9-2x,\ 0<x<6$ *(Inclined Straight line),* $V_A=+9,\ V_C=-3$

CD, $V_x=+9-2x6=-3$, $6<x<8$

$V_x=+9-2x6-3=-6$, $8<x<10$

Using these 3 equations and values determined above SFD is drawn.

<u>Bending Moment Calculations and BMD</u>

AC, $M_x=+9x-2x^2/2$, $0<x<6$ ----→1, $M_A=0$, $M_C=+18$ kN.m

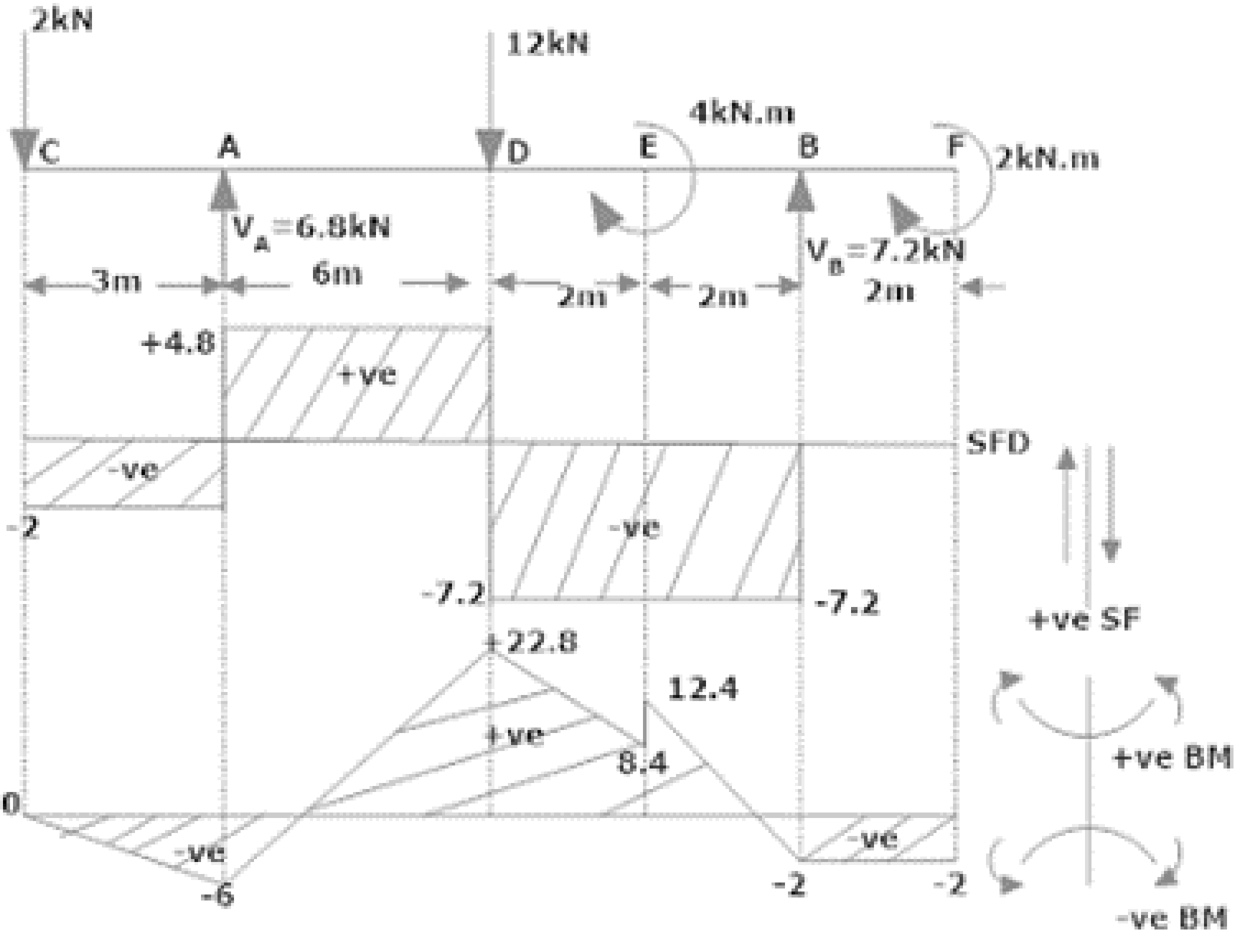
CD, $M_x = +9x-(2x6)(x-3)$, $6<x<9$ ----->2, $M_C=+18$ kN.m, $M_D=12$ kN.m

DB, Equation (3) not required, because $M_B=0$.

Maximum BM will occur at point of zero SF, ie. x=4.5m

& $M_{4.5}$ = 9x4.5-2x4.5x4.5/2= 20.25 kN.m.

Example 6 : Sketch SFD and BMD for the following overhanging beam loaded as Shown in the Figure.



<u>Reactions:</u>

Σ *M at A = 0,* ↻+ve

$-2x3+V_A x0+12x6+4-V_B x10+2=0$, *Therefore* $V_B=+7.2kN$

$\Sigma V=0$, ↑+ve,

$V_A + V_B - 2 - 12 = 0$, *Therefore* $V_A=+6.8kN$ ↑

<u>Shear Force Calculations & SFD</u>

Please <u>carefully connect</u> the numerical values with <u>Sign conventions</u> given on Right Hand Side.*Therefore* $_RV_C=-2$, $_LV_A=-2$, $_RV_A=+4.8$, $_LV_D=+4.8$, $_RV_D=-7.2$, $_LV_B=-7.2$, $_RV_B=0$

(explanation---------${}_{L}V_A$ means Shear Force at A on the Left hand side, ${}_{R}V_A$ means SF at A on Right hand side)

Plot these values at every point on left and right and connect these with straight lines- Result will be SFD

Bending Moment Calculations and BMD

Consider the Sign Convention as shown on RHS, calculate the values of BM at every point.

$M_C=0$, $M_A=-6kN.m$, $M_D=+22.8\ kN.m$, ${}_{L}M_E= +8.4kN.m$

${}_{R}M_E=+12.4\ kN.m$, $M_B=-2\ kN.m$, $M_F=-2.0\ kN.m$

Above **six examples** have been solved by regular procedure as given in all the books. In order to simplify the procedure, we may **omit the formation of equations** and using **"distinct features"** of Shearing Force and Bending Moment diagrams, the necessary **SFD** & **BMD** can be obtained by Mechanical Method.

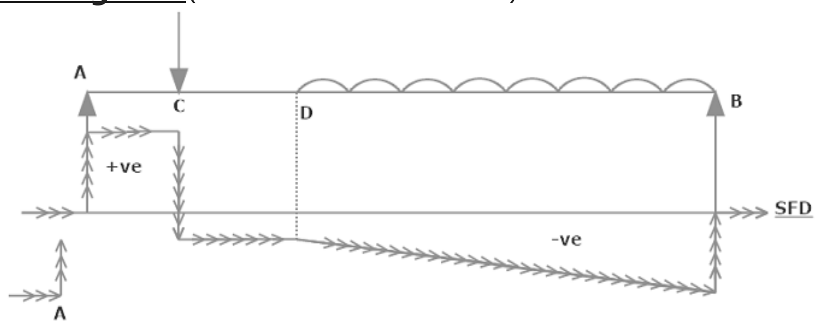
Distinct Features (or properties)

Shearing Force Diagram (SFD)

1. No load will be represented by a horizontal line in SFD.
2. A vertical load or a vertical reaction will be represented by a Vertical line in SFD.
3. Proceeding from left, uniformly distributed load will be represented by an inclined straight line sloping down in SFD

We will limit to only these 3 distinct features. Moment loading will not have any effect on SFD.

<u>Procedure of drawing SFD</u> (Just walk on the beam)



In the above diagram simple beam with loading is given. No numerical values are given. Just enter the beam at A and walk along the beam.

When you reach at A you will be lifted up by V_A Vertically upwards by the same magnitude. A to C, there is no load, as such A to C will be a horizontal line and one can walk straight to C.

At C, there is vertical down load and so it will push you down by same magnitude. Walking from C to D is similar to A to C. Accordingly CD will be a horizontal line. D to B is a uniformly distributed load, as per distinct feature 3, it will be inclined straight line sloping down.

Finally, when you come out of the beam, you will be pushed up by V_B to the level of the beam. If it does not, your calculations are wrong.

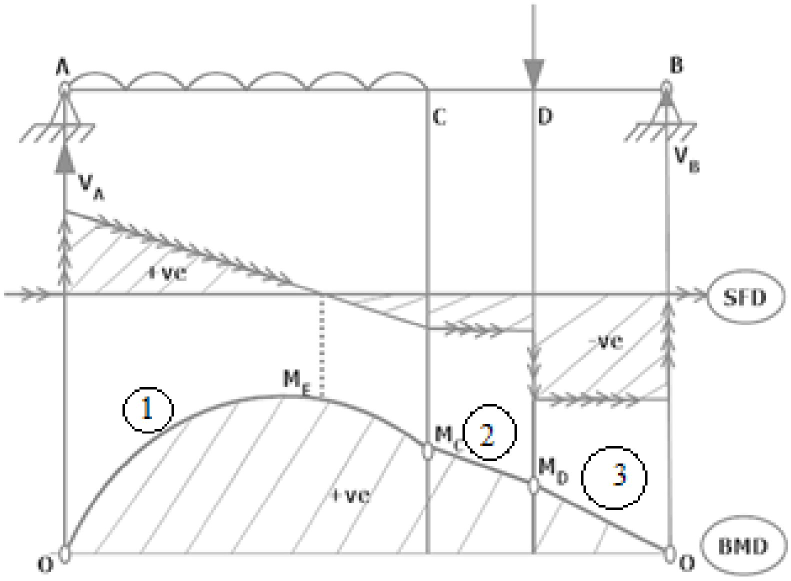
Please see the diagram carefully. The level at which you entered at A is same at which you got out at B.

<u>Distinct Features: Bending Moment Diagram (BMD)</u>

1. A moment (or couple) loading will be represented by a Vertical Line of that magnitude.
2. No load portions will be represented by straight lines (horizontal or inclined) in BMD.
3. Uniformly distributed load will be represented by a parabolic curve in BMD.
4. A sudden change in the slope of diagram in BMD will indicate presence of a vertical load or a vertical reaction at that point.

Only above four important features are given to solve most of the problems.

From 1st principles, calculate BM at principal locations A,B,C,D.



$M_A=0$,   Obvius

$M_B=0$,   Obvius

$M_C=+?$,   Calculate and mark the vertical ordinate in BMD as M_C

$M_D=+?$ Calculate and mark the vertical ordinate in BMD as M_D

C to D-No Load-----CD inclined straight line (draw line **2**)

D to B-No-Load-----DB inclined straight line (draw line **3**)

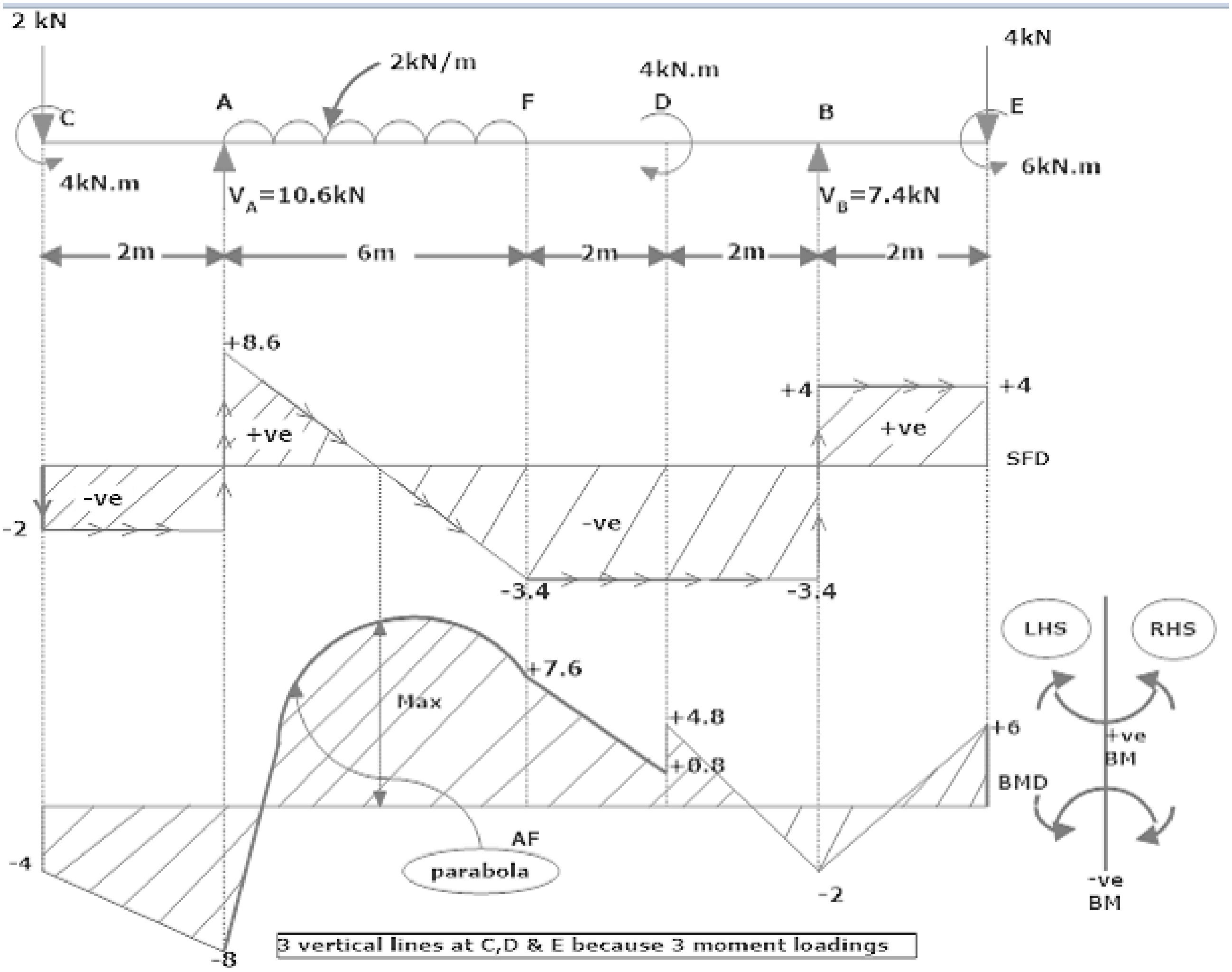
A to C ---------------A Parabolic Curve (draw curve **1)**

You need **3** points to sketch a curve, M_A, M_C and M_E.

$M_A=0$, M_C already calculated above, M_E=Maximum BM (A point at which Shearing Force is Zero)

Following Two Problems are solved by using *distinct features and mechanical method.*

Example No. 8
Using Distinct Features, Sketch SFD and BMD, giving values at principal locations.



Reactions:

$\Sigma M_{atA} = 0$, Clockwise +ve,

$-2 \times 2 - 4 + V_A \times 0 + (2 \times 6)\times(3) + 4 - V_B \times 10 + 4 \times 12 - 6 = 0$

$V_B = +7.4\ kN$

$\Sigma V = 0$, +ve,

$+V_A + V_B - 2 - (2 \times 6) - 4 = 0$ Therefore $V_A = +10.6\ kN$

Using Mechanical Method, SFD

i.e. Walking on the beam from C to E. SFD is drawn as shown.

BM Calculations at Principal Locations:

$M_C = -4$, $M_A = -8$,

$M_F = -4 - 2 \times 8 + 10.6 \times 6 - (2 \times 6) \times 3 = +7.6$ kN.m

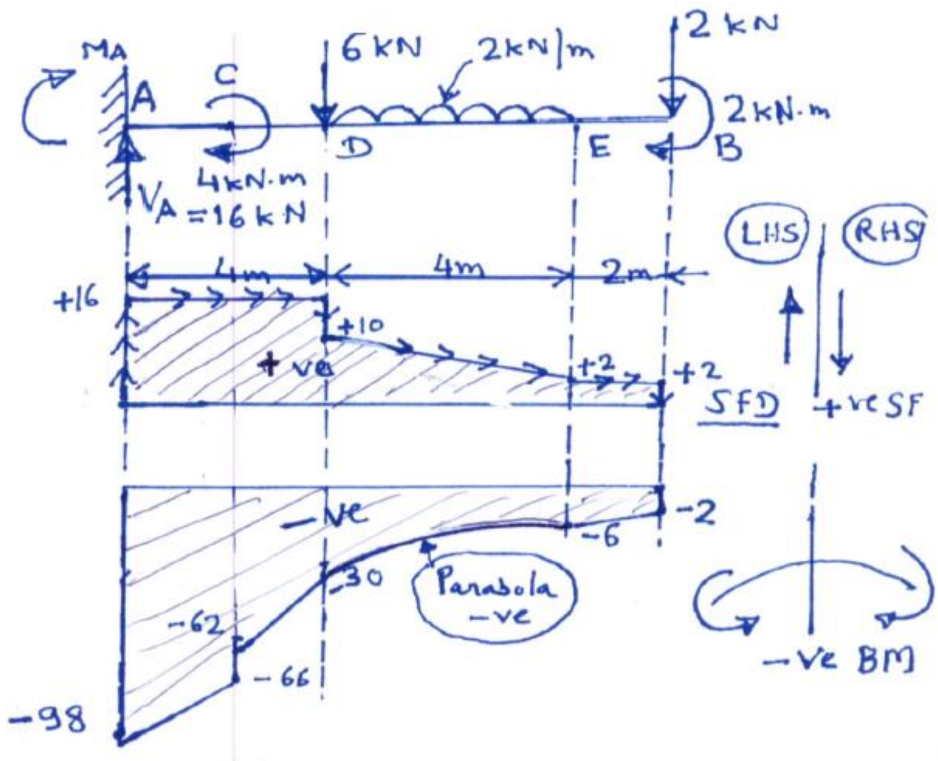
$M_E = +6$, $M_B = +6 - (4 \times 2) = -2$,

${}_RM_D = -4 \times 4 + 6 + 7.4 \times 2 = +4$ kN.m,

${}_LM_D = +4.8 - 4 = 0.8$ kN.m.

Example No : 9 A Cantilever beam problem

Using Distinct Features, Sketch SFD and BMD.



Reactions,

$\Sigma V = 0$, ↑+ve,

$V_A - 6 - 2 - 2 \times 4 = 0$, Therefore $V_A = +16$ kN

$\Sigma M_{atA} = 0$, Clockwise +ve , $M_A + V_A \times 0 + 6\times4 + 2\times4\times6 + 2\times10 + 2 = 0$

Therefore, $M_A = -98$ kN.m

$M_B = -2$ kN.m $\quad M_E = -2 - 2 * 2 = -6$ kN.m

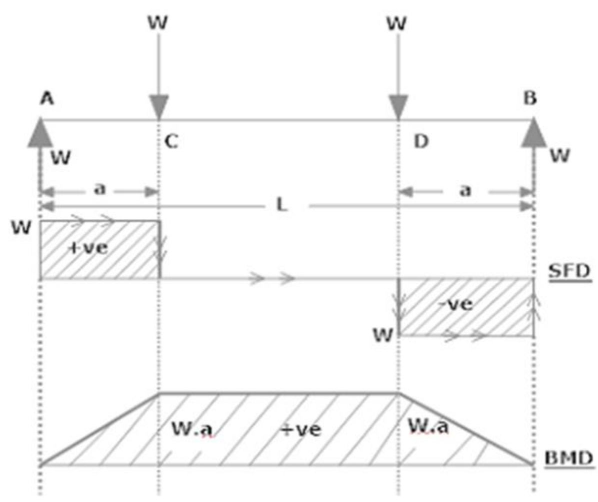
$M_D = -30$ kN.m $\quad {}_R M_C = -6 \times 2 - (2 \times 4)\, 4 - 2 \times 8 - 2 = -62$ kN.m

${}_L M_C = -66$ kN.m

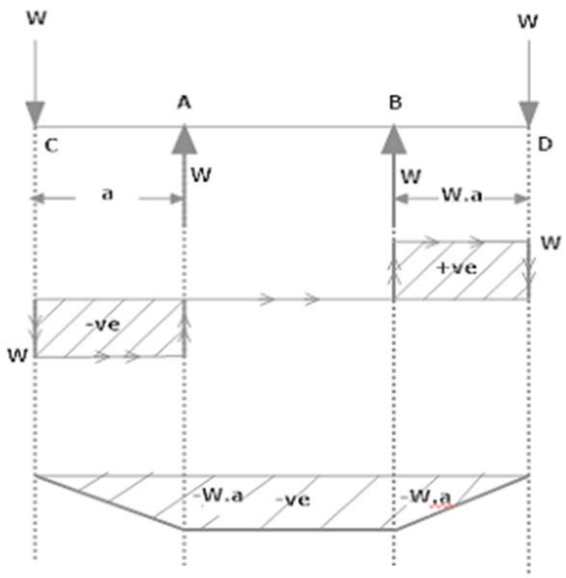
Cases of Pure Bending

A portion of the beam which does not carry any Shear Force, but has the bending moment, is called a case of Pure Bending.

Two such examples are shown below

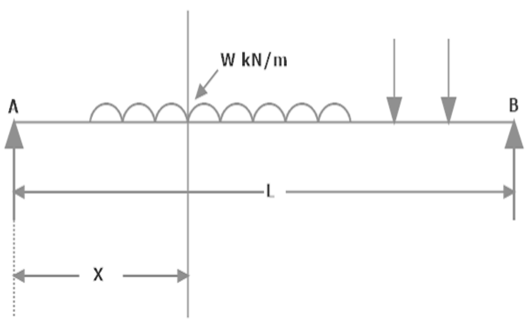


There is pure BM in portion CD.



There is pure BM in portion AB.

Relationship between "Load", "SF" & "BM"



Let the SF at section x from A be "V" and BM be "M",

The relationships will be as follows:

$$dV/dx = -w \quad \text{--------}\rightarrow (1)$$

$$dM/dx = V \quad \text{--------}\rightarrow (2)$$

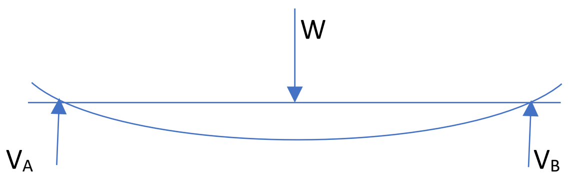
Therefore, $$d^2M/dx^2 = dV/dx = -w \quad \text{-----}\rightarrow (3)$$

Proof of these 3 equations will be given later after explaining the ***"Free Body Diagram"*** concept.

Define Point of Contraflexure

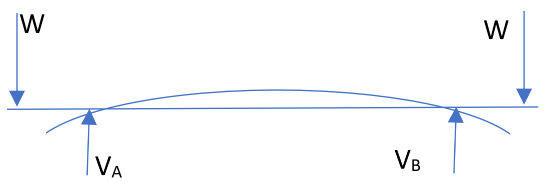
1. It is a point at which the curvature of the beam changes sign in the deflected shape. From concavity upwards to convexity upwards.
2. It is a point at which Bending Moment changes sign from positive to negative or from negative to positive.
3. It is a point at which Bending Moment is zero.

Following diagrams will explain the required definition:



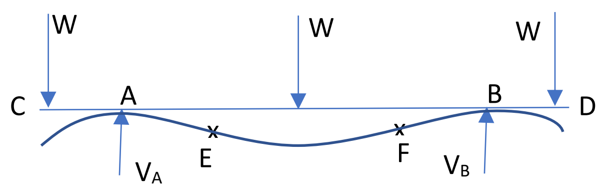
Uniform Curvature (+ve)

Concavity Upwards through out



Uniform Curvature (-ve)

Convexity Upwards through out



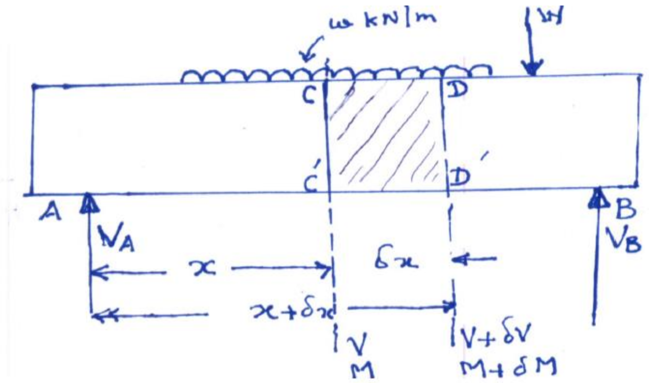
Both the types of curvatures appear.

The points (E & F) at which curvature changes sign are 2 points of contraflexure.

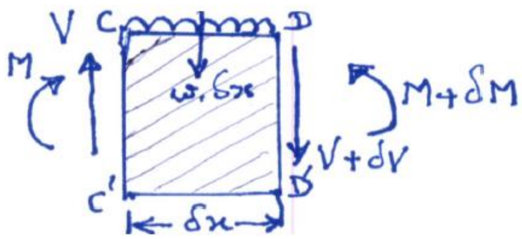
Free Body Diagram:

It is a diagram of an isolated portion of a structure which shows the applied loads and the loads exerted by the removed elements.

Relationship Between Load, Shear Force and Bending Moment using Free Body Diagram Concept:



Free Body Diagram of isolated portion CD:



FBD CDD'C'

Consider FBD as shown above.

$$\Sigma V = 0, \uparrow + ve, \qquad +V - w.\delta x - (V + \delta v) = 0$$

Therefore $\frac{dv}{dx} = -w$, in the limit $\delta x \to 0$, $\frac{dv}{dx} = -w$

$$\Sigma\, MatDD' = 0, \curvearrowright + ve, +M + V.\delta x - w.\frac{\delta x^2}{2} + (V + \delta v)0 - (M + \delta M) = 0$$

$$or, +V.\delta x - w.\frac{\delta x^2}{2} - \delta M = 0$$

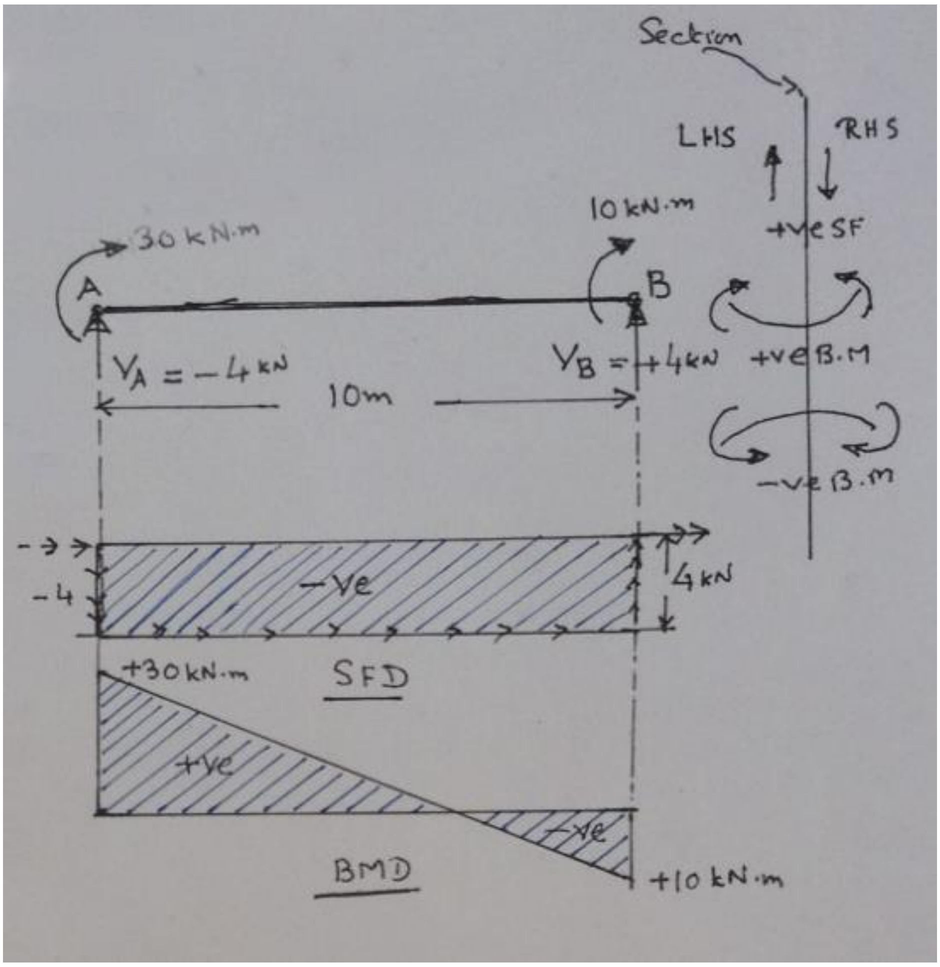
$$or, \frac{\delta M}{\delta x} = V - w.\frac{\delta x}{2}, In\ the\ limit\ \delta x \to 0, \qquad \frac{dM}{dx} = V$$

Differentiating 2, We get $\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w$

Equation 2 indicates that Maximum BM occurs when V=SF=0.

Additional Typical Problems
With only "Moment Loadings"

Example 1. Sketch SFD and BMD for the given beam with only Two Moment Loadings as shown. (M_A=+30 kN.m, M_B=+10 kN.m)



<u>**Reactions:** Apply conditions of Equilibrium</u>

Σ M at A= 0, clockwise +ve, $+30+V_A x0+10-V_B x10 = 0$

Therefore, V_B=+4 kN, and V_A = -4 kN

<u>**SFD** Just Walk on the Beam</u>

At A, go down by 4 kN . A to B no-load and as such Horizontal Straight line. At B go up by same amount V_B=4 and reach at the same level.

BMD M_A =+30 (See sign convention diagram on the right side)

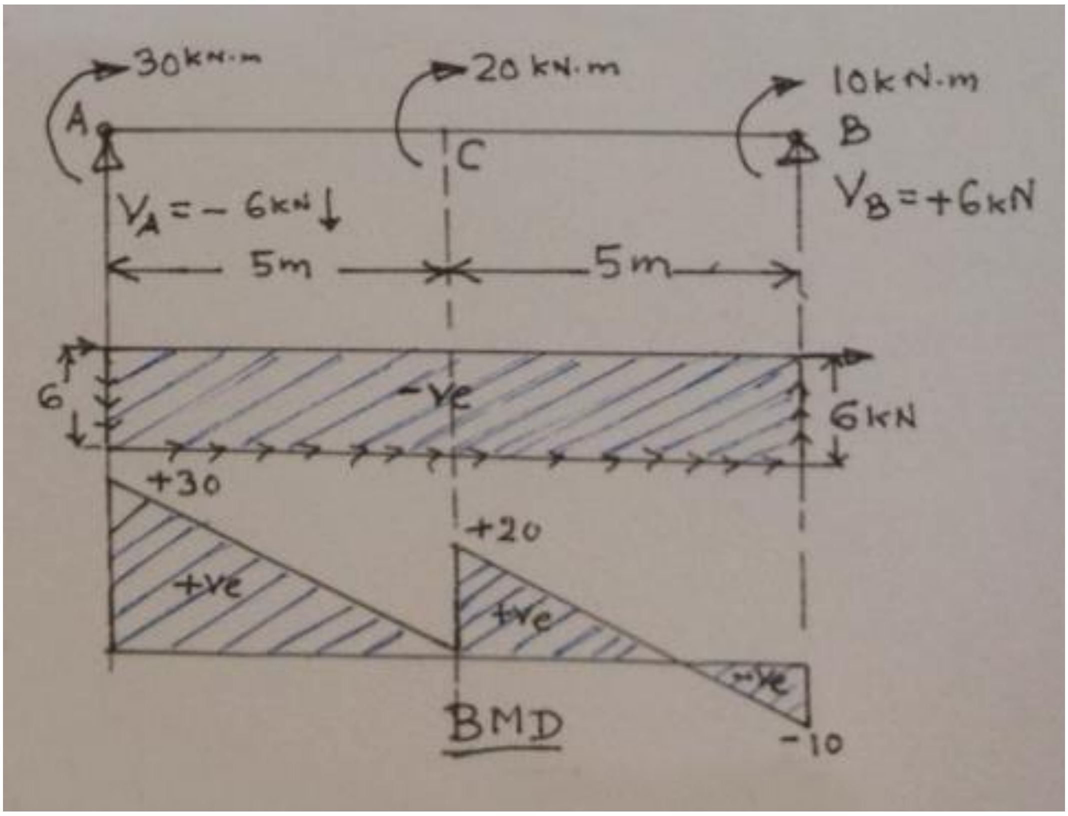
M_B =+30-4x10 = -10 kN.m

Or Look at the RHS of sign convention, clockwise, BM on

On RHS is -ve. i.e -10 kN.m

Also refer Distinct Feature 1 of BMD, Two Moment loadings are Represented by Two Vertical lines at A and B in BMD.

Example 2. Sketch SFD and BMDfor the given beam with Three Moment Loadings of 30 kN,m, 20 kN.m and10 kN.m as shown in the Figure (clockwise moments 0n LHS +ve)



Reactions Apply conditions of Equilibrium

M at A=0, Clockwise moments +ve,

+30+20+10+V_Ax0-V_Bx10=0 Therefore V_B=+6 kN

Hence, V_A=-6 kN

SFD Can be drawn mechanically as shown in the Figure

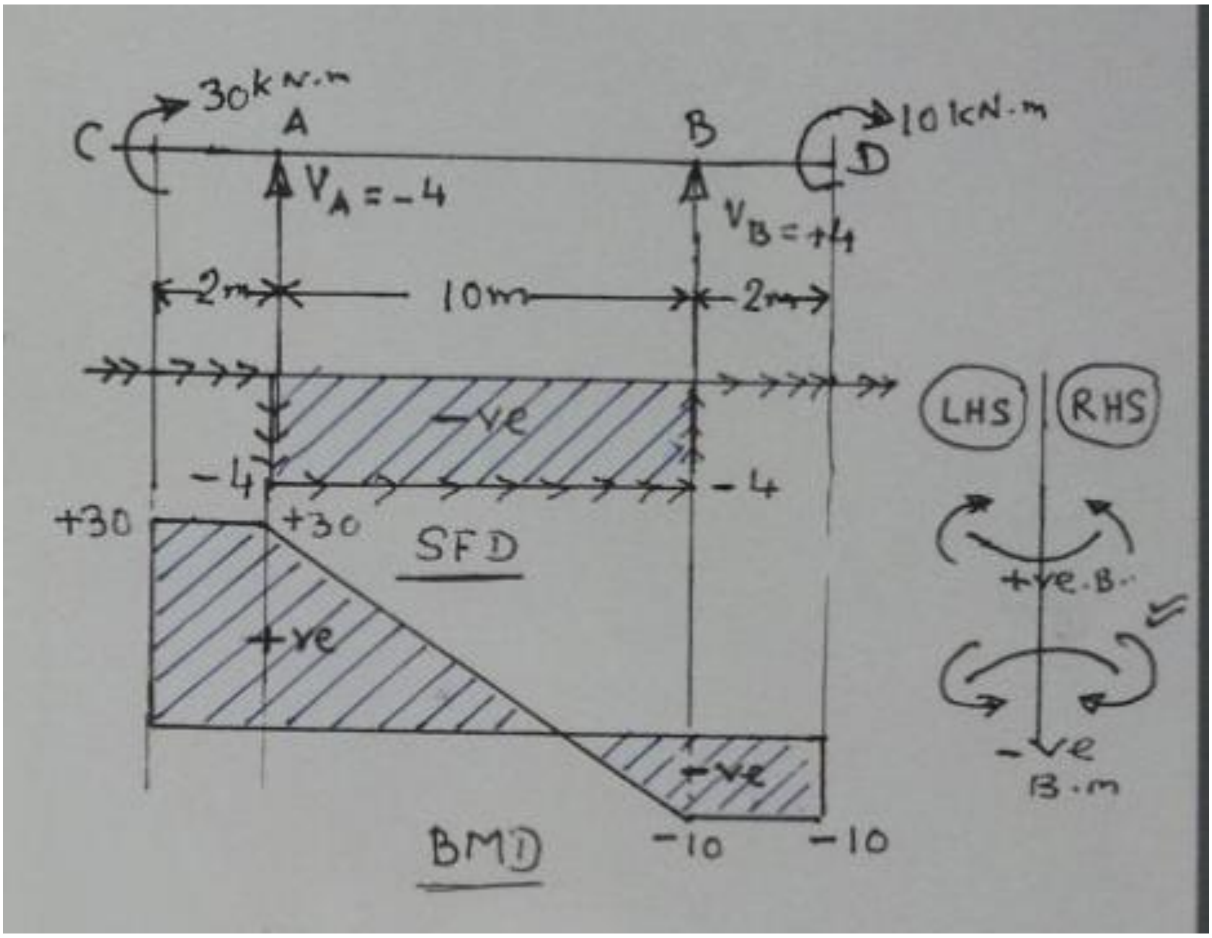
BMD Calculations

M_A = +30 kN.m, $_LM_C$ =+30 – 6 x 5=0, $_RM_C$ =+30 – 6 x 5 +20 = +20 kN.m

$_LM_B$=+30+20-6x10=-10 kN.m, $_RM_B$ = 10 Clockwise on RHS i.e. -10 kN.m

(As per adopted sign convention of BM)

Example 3 Sketch SFD and BMD for an overhanging beam with 2 Moment Loadings of 30 kN.m and 10 kN.m as shown.



Reactions **Apply conditions of Equilibrium.**

M at A=0, Clockwise +ve

$+30+V_A x0-V_B x10+10 = 0$, Therefore $V_B = +4$ kN, Hence, $V_A = -4$ kN

SFD Can be drawn Mechanically as shown (Just walk on the beam from C to D up and down at every location of vertical load or reaction)

BMD Calculations

$M_C = +30$ kN.m, $M_A = +30$ kN.m, $M_B = +30 - 4x10 = -10$ kN.m

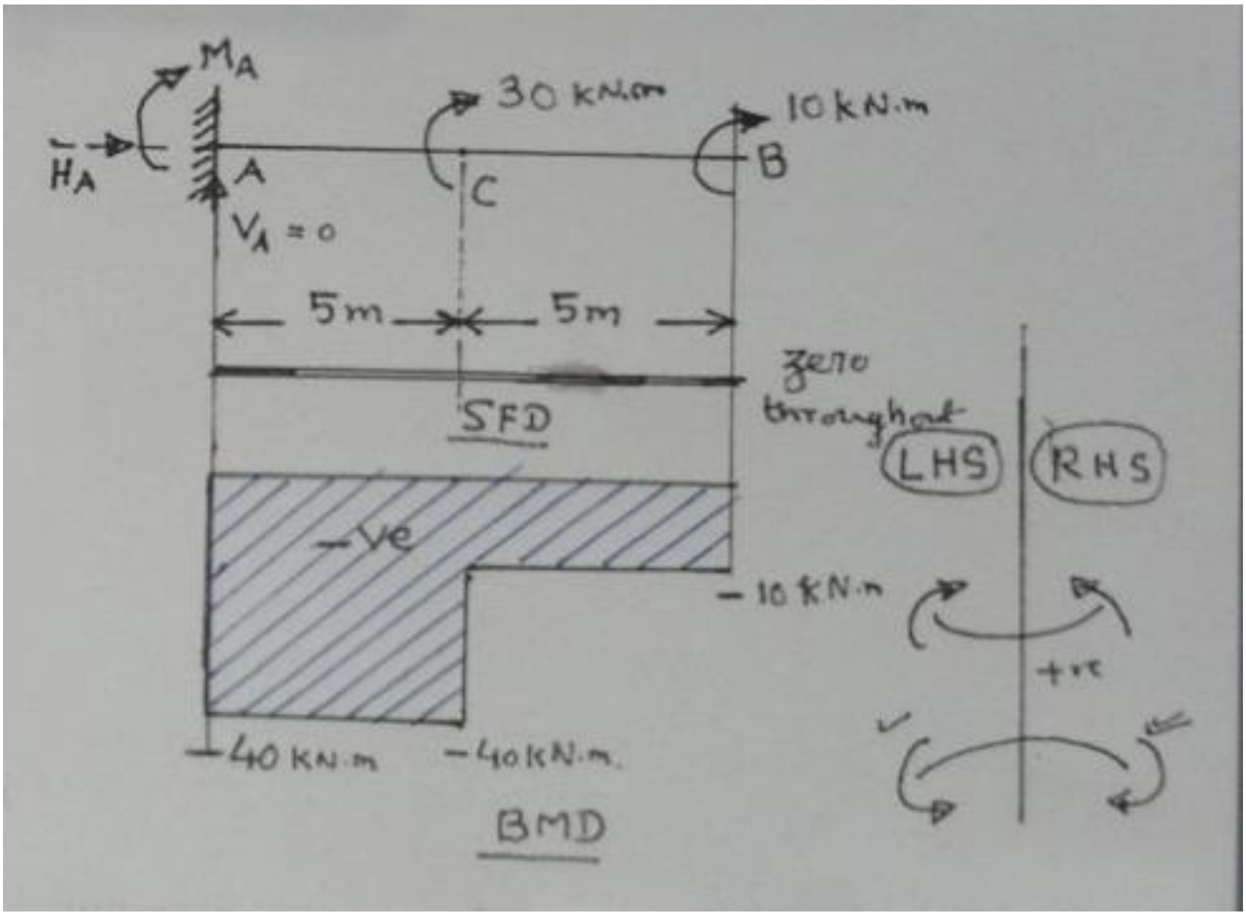
$M_D = +30+10 - V_A x12 + V_B x 2 = 30+10-4x12+4x2 = -10$ kN.m

(Please note that M_D=-10 kN.m is obvious from the definition of BM, considering RHS)

Two Moment Loadings (No Moment Reaction). Therefore,

Two Vertical Lines at C and D

Example 4. A Cantilever beam is subjected to 2 Moment loadings of 30 kN.m **and 10 kN.m as shown. Sketch SFD and BMD.**



Reactions **At rigid end "A", all the three movements are prevented and therefore, there will be three reactions, H ,V and M as shown in the the figure and are assumed to be +ve**

Apply conditions of Equilibrium, M at A = 0, Clockwise +ve,

$+M_A + H_A \times 0 + V_A \times 0 + 30 + 10 = 0$, Therefore $M_A = -40$ kN.m

SFD It is clear that both H_A and V_A will be Zero,

Hence, SFD will be Zero from A to B. A single horizontal straight line.

BMD <u>Calculations</u>

$M_A = -40$ kN.m, ${}_RM_C = -40+30 = -10$ kN.m, ${}_LM_B = -40+30 = -10$ kN.m

Important Conclusions:-

If only 3 distinct features of SF and 3 distinct features of BM are remembered, there will be no difficulty in drawing SFD and BMD. Students often make mistakes in drawing BMD when moment loading moment reactions are there. For this difficulty, only distinct feature No.1 of BM may be understood and remembered. Number of vertical lines in BMD will be equal to number of moment loadings and moment reactions given in the actual beam. In the above Example 4, it is seen that there are 3 vertical lines in BMD corresponding to 3 Moment loadings/Reactions.